Summary

For this section, I was going to derive the Hamilton-Jacobi (HJ) and Bellman equations, but due to time limitations and a lengthy discussion between Dr. Petrik and others, I only discussed the HJ equation.

I wrote the equations on the whiteboard and was about to explain from the first equation when Dr. Begum asked me, "How is the maximum invariant set related to the HJ equation?" It seems that I did not have a convincing answer to this question, and I plan to address it next week.

I began explaining the first equation and how the cost function is set up. Then, while discussing minimization, Dr. Petrik asked how we can ensure that this minimum exists. I responded that it depends on the lower and upper bounds of the cost function, but it became clear that we need to check the necessary and sufficient optimality conditions. The HJ equation also has necessary and sufficient optimality conditions. Another question he asked was about the input u(t), and I explained that u(t) belongs to U, which is a compact set. He also inquired whether u(t) must be continuous or if it can be discontinuous. I believe it must be continuous, but Dr. Yoon mentioned that in a Bang-Bang controller, u(t) is on or off. I am still unsure, but based on equations like integration and Taylor expansion, I believe that since u(t) is a function of the states, and the states are a function of time, and time is continuous, u(t) must be continuous. Moreover, what does "discontinuous" mean in this context?

I think we spent more time discussing the different parts of the cost function, identifying which parts depend on u(t) and which do not. What matters here is the HJ equation, a PDE, and to solve it, we need boundary conditions. Initially, the cost is split into two parts: terminal cost and the integral of the cost between different states. This seems to be due to the boundary conditions.

Additionally, Dr. Begum suggested changing the dynamics from an inverted pendulum to a simple car model to have a better perspective regarding safety. I also argued that we should now narrow down which approach we are pursuing, and Dr. Yoon mentioned that we still need to understand better how the HJ equation connects to the safety set.